SUBJECT: APPLIED MATHEMATICS-I
CODE: MTH301
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PROGRAMME: D.VOC. (INDUSTRIAL ELECTRONICS) &
D.VOC. (MECHANICAL-MANUFACTURING)

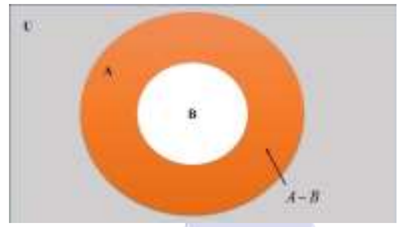
Source: NCERT, www.google.com

Previous Year Question Paper

2018 Regular (1st Semester)

Section-A (Objective types questions)

1. Draw Venn diagram for $A-B$ when $B \subset A$.



Solution: Given that $B \subset A$

$\Rightarrow$ Set B is completely inside the Set A

Now : $A-B=\{x : x \in A \text{ and } x \notin B\}$

The desired Venn diagram of given in the figure.

2. If $A \times B=\{(a,x),(a,y),(b,x),(b,y)\}$, Find A and B.

Solution: Given that $A \times B=\{(a,x),(a,y),(b,x),(b,y)\}$

We know that for Sets A and B the Cartesian product is given as, $A \times B=\{(a,b) : a \in A \text{ and } b \in B\}$

Therefore, Set $A=\{a,b\}, \quad B=\{x,y\}$

3. The 15th term of an A.P. is 34. Find the sum of its 29 terms.

Solution: Given that

$$T_{15}=34$$
$$\Rightarrow a+(15-1)d=34$$
$$\Rightarrow a+14d=34$$

Now: $S_n=\frac{n}{2}\left[2a+(n-1)d\right]$

$$\Rightarrow S_{29}=\frac{29}{2}\left[2a+(29-1)d\right]$$
$$\Rightarrow S_{29}=\frac{29}{2}\left[2a+(29-1)d\right]$$
$$\Rightarrow S_{29}=\frac{29}{2}\left[2\{a+14d\}\right]=\frac{29}{2}\left[2\times 34\right]$$
$$\Rightarrow S_{29}=986$$

4. Define the nth term of Harmonic Progression.

Solution: A harmonic progression is a sequence of real numbers formed by taking the reciprocals of an Arithmetic Progression.

Therefore, the nth term is $T_n=\dfrac{1}{a+(n-1)d}$

5. 4 boys and 3 girls are to be seated in 7 chairs, in how many ways can this be done if all boys are seated together?

Solution: Total 7 persons (4 boys and 3 girls) are to be seated in 7 chairs with all 4 boys are to be seated together. Therefore, we can count total 4 persons to be seated in 4 chairs and the numbers $={}^4P_4=4!$

Corresponding to each of these permutations, 4 boys can be seated together in total 4! ways.

Therefore, total required number of ways $=4!.4!.=576$

Source: NCERT, www.google.com

6. Find the number of subsets of the set $\{1,3,5,7,9,11,13.....23\}$ each having 3 elements.

Solution: $\text{Total no. of elemets in the set } \{1,3,5,7,9,11,13.....23\} = 12$

$$\text{So number of substes having 3 elements} = {}^{12}C_3 = \frac{12!}{3!.9!} = \frac{12.11.10}{3.2} = 220$$

7. If $\tan\theta = \frac{1}{2}$, what is the value of $\sin\theta$ and $\cos\theta$?

Solution: Given : $\tan\theta = \frac{1}{2}$

$$\Rightarrow \sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{1}{2}\right)^2 = 1 + \frac{1}{4} = \frac{5}{4}$$

$$\Rightarrow \sec\theta = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

$$\Rightarrow \cos\theta = \frac{1}{\sec\theta} = \frac{2}{\sqrt{5}}$$

$$\text{also, } \tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \sin\theta = \tan\theta \cdot \cos\theta = \left(\frac{1}{2}\right)\left(\frac{2}{\sqrt{5}}\right)$$

$$\Rightarrow \sin\theta = \left(\frac{1}{\sqrt{5}}\right)$$

8. Express $\sin 6\theta + \sin 4\theta$ as a product.

Solution: Formula: $\text{sinC+SinD} = 2\sin\left(\frac{\text{C+D}}{2}\right) \cdot \cos\left(\frac{\text{C}-\text{D}}{2}\right)$

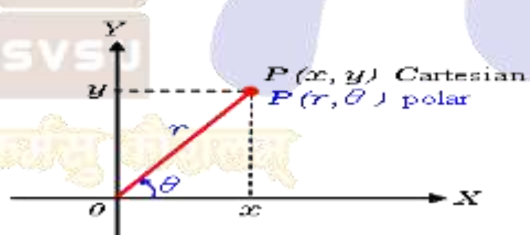
$$\Rightarrow \sin 6\theta + \sin 4\theta = 2\sin\left(\frac{6\theta + 4\theta}{2}\right) \cdot \cos\left(\frac{6\theta - 4\theta}{2}\right)$$

$$\Rightarrow \sin 6\theta + \sin 4\theta = 2\sin 5\theta \cdot \cos\theta$$

9. What is the relation between Cartesian and Polar Coordinates?

Solution:

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$\text{where, } r = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$



10. What is the slope of the line joining the points $A(6,8)$ and $B(4,14)$?

Solution: Given points: $(x_1, y_1) = (6,8)$ & $(x_2, y_2) = (4,14)$

$$\text{Formula: } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14-8}{4-6} = \frac{6}{-2} = -3$$

Section-B (Essay types questions)

1. Find the domain and range of the function $y=\sqrt{(3-x)(x-5)}$.

Solution: For domain:

$y=\sqrt{(3-x)(x-5)}$ is defined if

$(3-x)(x-5)\geq 0$

$\Rightarrow (x-3)(x-5)\leq 0$

$\Rightarrow 3\leq x\leq 5$

For range:

take $y=\sqrt{(3-x)(x-5)}$

$\Rightarrow y=\sqrt{-15+8x-x^2}=\sqrt{1-16+8x-x^2}$

$\Rightarrow y=\sqrt{1-\left(x^2-8x+16\right)}=\sqrt{1-(x-4)^2}$

$\Rightarrow 0\leq y\leq 1$

2. Three numbers are in A.P. The difference between the first and the last is 8 and the product of these two is 20. Find the numbers.

Solution: Let the numbers be $a, a+d, a+2d$

Therefore, $a-(a+2d)=8\Rightarrow -2d=8\Rightarrow d=-4$

and $a(a+2d)=20\Rightarrow a(a-8)=20$

$\Rightarrow a^2-8a-20=0$

$\Rightarrow a^2-10a+2a-20=0$

$\Rightarrow a(a-10)+2(a-10)=0$

$\Rightarrow (a+2)(a-10)=0$

$\Rightarrow a=-2,10$

$\Rightarrow$ Therefore the numbers are; $-2,-6,-10$ OR $10,6,2$

3. Find the sum of the sequence $8,88,888,.......$ up to n terms.

Solution: Let $S_n=8+88+888+.........n \text{ terms}$

$$\Rightarrow S_n=\frac{8}{9}(9+99+999+...........n \text{ terms})$$

$$\Rightarrow S_n=\frac{8}{9}\{(10-1)+(100-1)+(1000-1)+...........n \text{ terms}\}$$

$$\Rightarrow S_n=\frac{8}{9}\{(10+100+1000+.........n \text{ terms})-(1+1+1+..........n \text{ terms})\}$$

$$\Rightarrow S_n=\frac{8}{9}\left\{\frac{10\left(10^n-1\right)}{10-1}-n\right\}\quad\left(\text{Formula}: S_n=\frac{a\left(r^n-1\right)}{r-1}\right)$$

$$\Rightarrow S_n=\frac{8}{81}\left(10^{n+1}-9n-10\right)$$

4. Find the middle term(s) in the expression of $\left(\frac{3x}{4}-\frac{4y}{3}\right)^7$.

Solution: The general term : $T_{r+1}={}^{n}C_r X^{n-r}Y^r$

Given $n=7$

$\Rightarrow$ Total terms $=8$

$\Rightarrow$ 2 middle terms $=4^{th}$ and 5^{th}

$$\Rightarrow T_4=T_{3+1}={}^{7}C_3\left(\frac{3x}{4}\right)^{7-3}\left(-\frac{4y}{3}\right)^3=\frac{7!}{3!.4!}\left(\frac{81x^4}{256}\right)\left(\frac{-64y^3}{27}\right)=-\frac{105}{4}x^4y^3$$

$$\Rightarrow T_5=T_{4+1}={}^{7}C_4\left(\frac{3x}{4}\right)^{7-4}\left(-\frac{4y}{3}\right)^4=\frac{7!}{4!.3!}\left(\frac{27x^3}{64}\right)\left(\frac{256y^4}{81}\right)=\frac{140}{3}x^3y^4$$

5. Solve for partial fraction of $\frac{2x-4}{(x+1)^2(x-3)}$.

Solution:

$$\frac{2x-4}{(x+1)^2(x-3)}=\frac{A}{(x+1)}+\frac{B}{(x+1)^2}+\frac{C}{(x-3)}\text{ ....(1)}$$

$$\Rightarrow \frac{2x-4}{(x+1)^2(x-3)}=\frac{A(x+1)(x-3)+B(x-3)+C(x+1)^2}{(x+1)^2(x-3)}$$

$$\Rightarrow 2x-4=A(x+1)(x-3)+B(x-3)+C(x+1)^2\text{ ....(2)}$$

Put $x+1=0\Rightarrow x=-1$ in (2)

$$\Rightarrow 2(-1)-4=A(0)(-1-3)+B(-1-3)+C(0)^2$$

$$\Rightarrow -6=B(-4)\Rightarrow B=\frac{3}{2}$$

Put $x-3=0\Rightarrow x=3$ in (2)

$$\Rightarrow 2(3)-4=A(3+1)(0)+B(0)+C(3+1)^2$$

$$\Rightarrow 2=16C\Rightarrow C=\frac{1}{8}$$

Put $x=0$ in (2)

$$\Rightarrow 2(0)-4=A(0+1)(0-3)+B(0-3)+C(0+1)^2$$

$$\Rightarrow -4=-3A-3B+C$$

$$\Rightarrow -4=-3A-3\left(\frac{3}{2}\right)+\frac{1}{8}\Rightarrow A=-\frac{1}{8}$$

Putting the values of A,B,C in (1) we get :

$$\frac{2x-4}{(x+1)^2(x-3)}=\frac{-1}{8(x+1)}+\frac{3}{2(x+1)^2}+\frac{1}{8(x-3)}$$

Source: NCERT, www.google.com

6. Prove $\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{6}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)=\frac{1}{16}$

Solution:

$$\text{L.H.S.}=\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{5\pi}{6}\right)\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)$$

$$=\sin\left(\frac{\pi}{18}\right)\sin\left(\pi-\frac{\pi}{6}\right)\frac{1}{2}\left\{2\sin\left(\frac{5\pi}{18}\right)\sin\left(\frac{7\pi}{18}\right)\right\}$$

$$=\frac{1}{2}\sin\left(\frac{\pi}{18}\right)\sin\left(\frac{\pi}{6}\right)\left\{\cos\left(\frac{5\pi}{18}-\frac{7\pi}{18}\right)-\cos\left(\frac{5\pi}{18}+\frac{7\pi}{18}\right)\right\}\quad\left\{\because \sin(\pi-\theta)=\sin\theta, 2\sin A\sin B=\cos(A-B)-\cos(A+B)\right\}$$

$$=\frac{1}{4}\sin\left(\frac{\pi}{18}\right)\left\{\cos\left(\frac{-2\pi}{18}\right)-\cos\left(\frac{12\pi}{18}\right)\right\}\quad\left\{\because \sin\left(\frac{\pi}{6}\right)=\frac{1}{2}\right\}$$

$$=\frac{1}{4}\sin\left(\frac{\pi}{18}\right)\left\{\cos\left(\frac{2\pi}{18}\right)-\cos\left(\frac{2\pi}{3}\right)\right\}\quad\left\{\because \cos(-\theta)=\cos\theta\right\}$$

$$=\frac{1}{4}\sin\left(\frac{\pi}{18}\right)\left\{\cos\left(\frac{2\pi}{18}\right)-\left(-\frac{1}{2}\right)\right\}\quad\left\{\because \cos\left(\frac{2\pi}{3}\right)=-\frac{1}{2}\right\}$$

$$=\frac{1}{4}\left\{\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{2\pi}{18}\right)+\frac{1}{2}\sin\left(\frac{\pi}{18}\right)\right\}$$

$$=\frac{1}{8}\left\{2\sin\left(\frac{\pi}{18}\right)\cos\left(\frac{2\pi}{18}\right)+\sin\left(\frac{\pi}{18}\right)\right\}$$

$$=\frac{1}{8}\left\{\sin\left(\frac{\pi}{18}+\frac{2\pi}{18}\right)+\sin\left(\frac{\pi}{18}-\frac{2\pi}{18}\right)+\sin\left(\frac{\pi}{18}\right)\right\}\quad\left\{\because 2\sin A\cos B=\sin(A+B)+\sin(A-B)\right\}$$

$$=\frac{1}{8}\left\{\sin\left(\frac{3\pi}{18}\right)+\sin\left(\frac{-\pi}{18}\right)+\sin\left(\frac{\pi}{18}\right)\right\}$$

$$=\frac{1}{8}\left\{\sin\left(\frac{\pi}{6}\right)-\sin\left(\frac{\pi}{18}\right)+\sin\left(\frac{\pi}{18}\right)\right\}\left\{\because \sin(-\theta)=-\sin\theta\right\}$$

$$=\frac{1}{8}.\frac{1}{2}=\frac{1}{16}=\text{R.H.S}$$

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7. Find the perpendicular distance of the point $(1,2)$ on the straight lines $3x+7y+14=0$.

Solution: Given point $(x_0,y_0)=(1,2)$ and line $3x+7y+14=0$

Comparing with general equation of line $Ax+By+C=0$, we have: $A=3, B=7, C=14$

Formula: $d=\frac{|Ax_0+By_0+C|}{\sqrt{A^2+B^2}}$

$$\Rightarrow d=\frac{|3\times1+7\times2+14|}{\sqrt{3^2+7^2}}\Rightarrow d=\frac{|31|}{\sqrt{58}}\quad\Rightarrow d=\frac{31}{\sqrt{58}}\text{ units}$$

8. Determine x so that 4 is the slope of the line through the points $A(6,12)$ and $B(x,8)$.

Solution: Given Points: $(x_1,y_1)=(6,12)$, $(x_2,y_2)=(x,8)$

Formula: $m=\frac{y_2-y_1}{x_2-x_1}\Rightarrow 4=\frac{8-12}{x-6}$

$$\Rightarrow 4=\frac{-4}{x-6}\Rightarrow 1=\frac{-1}{x-6}\Rightarrow x-6=-1\Rightarrow x=5$$

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